

# The Metallicity Dependence of RR Lyrae Absolute Magnitudes from Synthetic Horizontal-Branch Models

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## ABSTRACT

A grid of synthetic horizontal-branch (SHB) models based on HB evolutionary tracks with improved physics has been constructed to reconsider the theoretical calibration of the dependence of  $M_v(RR)$  on metallicity in globular clusters, and the slope of the mean  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  relation. The SHB models confirm Lee's earlier finding (Lee 1991) that the slope of the  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  relation is itself a function of the metallicity range considered (see also Caputo 1997), and that in addition, for a given  $[\text{Fe}/\text{H}]$ , RR Lyrae luminosities depend on HB morphology. This is due to the fact that HB stars pass through the RR Lyrae instability strip at different evolutionary stages, depending on their original position on the HB. At  $[\text{Fe}/\text{H}] = -1.9$ , and for HB type 0, the models yield  $M_V(RR) = 0.47 \pm 0.10$ . The mean slope for the zero-age HB models is 0.204. Since there is no simple universal relation between  $M_v(RR)$  and metallicity that is applicable to all globular clusters, the HB morphology of each individual cluster must be taken into account, in addition to  $[\text{Fe}/\text{H}]$ , in deriving the appropriate  $M_v(RR)$ . Taking HB morphology into account, we find that the slope of the mean  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  relation varies between 0.36 for the clusters with galactocentric distances  $R_{gc}$  less than 6 kpc and 0.22 for clusters with  $6 < R_{gc} \leq 20 \text{ kpc}$ . Implications for interpreting observations of field RR Lyrae variables and for absolute globular cluster ages and galactic chronology are briefly discussed.

*Subject headings:* Galaxy: formation - Galaxy: halo - globular clusters: general - stars: evolution - stars: horizontal branch - stars: RR Lyrae

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## 1. Introduction

It has long been realized that determination of the absolute ages of globular star clusters in our Galaxy is most vulnerable to the uncertainty in their distances (see e.g. Renzini 1991). Because of the difficulty in determining the precise position of the unevolved main sequence in globular clusters, the RR Lyrae variables have been in recent years the preferred distance indicators for globular clusters, and used in the calibration of the  $\Delta V$  method for comparing observed CMD's to theoretical isochrones (Renzini 1991; Buonanno et al. 1994). In this method, the quantity  $\Delta V$  is defined as the difference in  $V$  magnitude between the main sequence turnoff bluest point and the HB at the same color in the cluster CMD. This approach has the advantage of being independent of distance and interstellar reddening. The quantity  $\Delta V$  can in principle be calibrated with theoretical isochrones since it relies primarily on the physics of the deep interior, which is believed to be relatively well-known, and less on uncertain assumptions about the efficiency of convection in the envelope and on color transformations (see e.g. the discussion of model uncertainties by Chaboyer et al. 1996a).

The purpose of this paper is to present a new theoretical calibration of the HB luminosity based on updated stellar evolutionary sequences and synthetic HB (SHB) models, and to analyze the sensitivity of  $M_V(RR)$  to  $[\text{Fe}/\text{H}]$ . Other parameters, such as HB morphology and input parameters used in the construction of SHBs, also affect the relation. The slope of the  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  relation is needed to discuss the relative ages of globular clusters and to determine whether there exists an age-metallicity correlation in the Galactic halo (Zinn 1985, 1993; Chaboyer, Demarque & Sarajedini 1996). Both the slope and zero-point (and their uncertainties) are also essential in discussions of the absolute ages of the globular clusters (Chaboyer et al. 1996, 1998; Demarque 1997).

With the first parallaxes for field subdwarfs from the Hipparcos satellite now becoming available, there has been a flurry of new interest in using the main sequence to derive globular cluster distances independently from the RR Lyrae luminosity calibration (Reid 1997; Gratton et al. 1997; Salaris, Degl'Innocenti & Weiss 1997). In early studies, the distance modulus of a globular cluster was derived by fitting the cluster main sequence to a main sequence derived from field subdwarfs with the same metallicity as the cluster (e.g. see the review by Sandage 1986). But because the uncertainties in trigonometric parallaxes for field subdwarfs were large, and the photometry of globular cluster main sequence stars was very uncertain, this approach had generally been abandoned in recent years. Preliminary analyses of the Hipparcos data indicate that the subdwarfs are intrinsically more luminous than previously believed, and therefore that globular cluster distances have been underestimated in the past. If correct, this means a more luminous HB and a higher luminosity for the RR Lyrae variables. In view of the crucial importance of globular cluster distances in understanding the evolution of galaxies and cosmology, new theoretical SHB models have been constructed.

## 2. Evolutionary Tracks

The evolutionary sequences presented here are quite similar to the many other HB models in the recent literature (Sweigart 1987; Lee & Demarque 1990; Dorman, Lee & Vandenberg 1991; Dorman 1992; Yi, Lee & Demarque 1993; Castellani et al. 1994; Caputo & Degl’Innocenti 1995; Mazzitelli, D’Antona & Caloi 1995), except for recent updates in the opacities and equation of state (Rodgers 1986; Iglesias & Rodgers 1996; Rodgers, Swenson & Iglesias 1996).

The evolutionary sequences are adopted from Yi, Demarque & Kim (1997) but extended to a finer grids of metallicity and mass, i.e. in the range  $0.52 - 0.92M_{\odot}$  at  $0.02M_{\odot}$  intervals, for the following metallicities:  $Z = 0.0001, 0.0002, 0.0004, 0.0007, 0.001, 0.002$ , and  $0.004$ , corresponding to  $[\text{Fe}/\text{H}]$  between  $-2.3$  and  $-0.7$ . The helium content by mass  $Y$  was taken from the evolutionary tracks that were used for the new Yale Isochrones<sup>2</sup>, corresponding to an initial  $Y = 0.23$ . The main difference in the input physics with the Lee & Demarque (1990) models is the introduction of the OPAL opacities and equation of state (Iglesias & Rodgers 1996; Rodgers, Swenson & Iglesias 1996). As a result, for  $Y_{MS} = 0.23$ , the current HB models of Yi et al. (1997) used in this study are approximately  $0.05 - 0.1$  mag. fainter than those of Lee & Demarque (1990). The Yi et al. models are approximately  $0.02 - 0.05$  mag. fainter than those of Caloi et al. (1997) and Cassisi et al. (1998) on the zero-age HB (ZAHB) in the instability strip at  $[\text{Fe}/\text{H}] = -2$ . This difference seems to be caused mostly by the fact that the helium core masses of the Yi et al. models are smaller, by  $2 - 5\%$ , than the core masses of the Caloi et al. and Cassisi et al. models. Table 1 lists the helium abundances in the envelope and the helium core masses for given metallicities used in the Yi et al HB models. These values are obtained from stellar models at the onset of helium ignition at the tip of the giant branch, which included the effects of the OPAL opacities, and the same equation of state as in Guenther et al. (1992).

## 3. Synthetic Horizontal-Branch (SHB) Models

The SHB models were derived using the technique introduced by Rood (1973), and extended by Lee et al. (1990). The mass distribution on the HB is defined by the following truncated Gaussian distribution:

$$\Psi(M) = \Psi_0 [M - (\overline{M}_{HB} - \Delta M)] (M_{RG} - M) \exp\left[-\frac{(\overline{M}_{HB} - M)^2}{2\sigma^2}\right] \quad (1)$$

where  $\sigma$  is a mass dispersion factor in solar mass,  $\Psi_0$  is a normalization factor, and  $\overline{M}_{HB}$  ( $\equiv M_{RG} - \Delta M$ ) is the mean mass of HB stars. Three values of  $\sigma$  (0.01, 0.02 and 0.03 solar masses) have been chosen, where the preferred value  $\sigma = 0.02$  is the mean of a representative group of clusters (see Table 1 in Lee 1990), and 0.01 and 0.03 were added to illustrate the sensitivity of the SHB models to the choice of  $\sigma$ .

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<sup>2</sup>See Yi et al. at <http://www.shemesh.gsfc.nasa.gov/> for both HB tracks and isochrones used in this study.

When considering the properties of stars in the RR Lyrae instability strip, it is convenient to introduce the HB Type index, which is defined as the ratio  $(B-R)/(B+V+R)$ , where B, V, and R are the numbers of blue HB stars, RR Lyrae variables, and red HB stars, respectively. This parameter is convenient in classifying HB morphology; we note that  $(B-R)/(B+V+R)$  ranges from  $-1$ , for clusters that display only a red HB (e.g. 47 Tuc), to  $+1$ , for clusters containing only blue HB stars (e.g. NGC 6752). Clusters that contain nearly equal numbers of red and blue HB stars (e.g. M3), are assigned values of  $(B-R)/(B+V+R)$  near 0. One of the advantages of this morphology index is that it includes the RR Lyrae variables, thus distinguishing between two clusters with the same numbers of blue or red HB stars, but differing in their RR Lyrae populations.

Figure 1 illustrates the dependence of the mean RR Lyrae visual absolute magnitude  $\langle M_V(RR) \rangle$  on metallicity and on HB Type, based on the SHB models constructed with  $\sigma = 0.02$ . The colors and bolometric corrections were taken from Green et al. (1987). As discussed earlier, the use of improved physics reduces the luminosities of SHBs for a given helium abundance. At  $[\text{Fe}/\text{H}] = -1.9$ , and for HB type 0, the models yield  $M_V(RR) = 0.47 \pm 0.10$ . This result is consistent with Walker’s (1992) value for the LMC RR Lyrae variables which is based on the classical cepheid distance scale, and with the distance of SN1987a (Panagia et al. 1991; Gould 1995; Sonneborn 1997).

Using observational data, Fusi Pecci et al. (1992) concluded that  $(B-R)/(B+V+R)$  depends primarily on the location of the peak of the color distribution of the HB, and only slightly on the dispersion in color of the HB stars. This conclusion is confirmed by the theoretical SHB models. For a fixed metallicity, the run of  $M_V(RR)$  predicted by the SHB models as a function of HB Type is found to depend weakly on the choice of the mass dispersion  $\sigma$  in eq. (1). This is illustrated in Figure 2 where RR Lyrae magnitudes for two extreme values of  $\sigma$ , 0.01 and 0.03, are plotted. The peculiar behavior near HB type 0.9 is caused by the particular metallicity dependence of the vertical width of the HB tracks. This vertical width is narrower for  $Z = 0.0002$  than for  $Z = 0.0004$ . Thus although the ZAHB luminosity is brighter for  $Z = 0.0002$ , the evolved RR Lyrae variables of  $Z = 0.0004$ , which are near the end of their HB tracks, could be brighter than those for  $Z = 0.0002$ . Increasing  $\sigma$  will dilute this effect. As a result, we do not see this behavior at larger  $\sigma$  in Figure 2.

#### 4. Is there a Universal Slope to the $\langle M_V(RR) \rangle$ - $[\text{Fe}/\text{H}]$ Relation?

The dependence of  $\langle M_V(RR) \rangle$  on  $[\text{Fe}/\text{H}]$  is needed to derive the relative ages of globular clusters of different metallicities. It is critical in deriving the chronology of the Galactic halo, its chemical enrichment, and in particular the possible existence of an age-metallicity correlation in the halo. It is customary to assume a linear relation between  $M_V(RR)$  and  $[\text{Fe}/\text{H}]$ , i.e. to write:

$$M_V(RR) = \mu [Fe/H] + \gamma \quad (2)$$

where, when used for globular cluster dating, the slope  $\mu$  affects the relative ages of clusters of different metallicities, and both  $\mu$  and  $\gamma$  determine the absolute ages.

There has been much debate about the value of the slope  $\mu$  over the years. Recently, from an analysis of the Oosterhoof-Sawyer period shift effect in globular clusters, Sandage (1993) has derived a “steep”  $\mu = 0.30 \pm 0.12$ , while studies based on the Baade-Wesselink method of determining the absolute magnitude of variable stars have yielded a “shallow” slope  $\mu$ , in the vicinity of 0.20 or less; Jones et al. (1992) derived  $\mu = 0.16 \pm 0.03$  and Skillen et al. (1993) determined  $\mu = 0.21 \pm 0.05$ . Recent analyses of these data by Sarajedini et al. (1997) and by Fernley et al. (1998) yielded  $\mu = 0.22 \pm 0.05$  and  $0.20 \pm 0.04$ , respectively. HST observations of the HB luminosity of three clusters in M31 (by Ajhar et al. 1996) have yielded a very shallow value of  $\mu = 0.08 \pm 0.13$ . Also using HST observations of the CMD’s of eight globular clusters in M31, Fusi Pecci et al. (1996) derived  $\langle M_V \rangle = (0.13 \pm 0.07)[\text{Fe}/\text{H}] + (0.95 \pm 0.09)$  for the mean magnitude of the HB in the instability strip. Theoretical estimates have consistently yielded  $\mu$  values in the range 0.18-0.20 for models near the ZAHB (Lee et al. 1990; Salaris et al. 1997).

Discussions of the theoretical  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  relation are frequently made using ZAHB models (e.g., Caloi et al. 1997). Sometimes an evolutionary correction is applied to take into account the fact that RR Lyrae variables are not observed in their original ZAHB position, and have evolved both in color and magnitude (Carney et al. 1992). Synthetic HB models are needed to provide a realistic description of these evolutionary corrections, which are found to differ significantly depending on the mass and the chemical composition of the models. Furthermore, only with SHB models is it possible to evaluate the effects of HB morphology on the value of  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  for a given metallicity, as was done originally by Lee (1991).

In his study of the RR Lyrae luminosities in  $\omega$  Cen, Lee (1991) pointed out that  $M_V(RR)$  is not a unique function of metallicity, particularly at the lowest metallicities. The effect is particularly marked for clusters with very blue stars on the HB, corresponding to HB Types approaching +1. Figure 1 updates the original Lee calibration. It is clear from Figure 1 that there is nothing universal about the value of  $\mu$ , and great caution should be used when applying equation (2) to derive RR Lyrae magnitudes without taking into account the HB morphology type of the population to which they belong (see also Caputo 1997).

To examine this point in more detail, let us consider first the hypothetical case where the HB is evenly populated with stars over a wide range in  $[\text{Fe}/\text{H}]$ . Since there is very small variation in  $\langle M_V(RR) \rangle$  over the range in HB type -0.5 to +0.5 (see Figs. 1 and 2), we may use the calculations for HB type = 0.0 to approximate this case. The resulting relationship between  $\langle M_V(RR) \rangle$  and  $[\text{Fe}/\text{H}]$  is shown in Figure 3, where one can see that there are small variations in slope over narrow ranges in  $[\text{Fe}/\text{H}]$  (see also Caputo 1997). The slopes found from our calculations are listed in Table 2. Since most observational studies have considered variables that span a wide range in  $[\text{Fe}/\text{H}]$ , e.g., -2.2 to -0.5, it is the slope over a wide range that is of most interest. While the relationship given by our calculations is non-linear, the departures from a straight-line

fit are small (see Figure 3) and would be very hard to detect observationally. The slope of the line in Figure 3 (0.21) is similar to that given by ZAHB calculations, but the zero-point (0.89) is somewhat brighter because the RR Lyrae variables have evolved from the ZAHB. It is expected that this  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  relationship will not apply to all stellar populations because HB morphology changes with  $[\text{Fe}/\text{H}]$ , the so-called first parameter, and also varies at constant  $[\text{Fe}/\text{H}]$ , the second parameter effect. This is most easily illustrated by considering the relationships expected for the HB morphologies of the globular clusters lying in different radial zones in the Milky Way.

Previous investigations (e.g., Searle & Zinn 1978, Lee et al. 1994) have shown that the globular clusters of the inner halo and bulge exhibit a tight relationship between HB morphology and  $[\text{Fe}/\text{H}]$ , which may mean that the second parameter effect is absent or weak among these clusters. The top graph in Figure 4 shows this relationship for the globular clusters that have galactocentric distances,  $R_{gc} \leq 6 \text{ kpc}$  (data from the 1999 June 22 revision of the Harris 1996 catalogue). To derive the  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  relationship for this group of clusters, we estimated the mean HB type of the clusters at the metallicities of our calculations and then derived  $M_V(RR)$  from the synthetic HB for that HB type and  $[\text{Fe}/\text{H}]$ . When estimating the mean HB type, lower weight was given to the clusters with HB type  $\sim 1.0$ , because these very blue HB clusters contain few, if any, RR Lyrae variables. To obtain additional points, we interpolated in  $[\text{Fe}/\text{H}]$  midway between the values of our calculations. Figure 4 shows the mean HB types (x's) used in this procedure and the resulting values of  $\langle M_V(RR) \rangle$  are plotted against  $[\text{Fe}/\text{H}]$  in the top diagram of Figure 5. Because the globular clusters with  $[\text{Fe}/\text{H}] < -1.5$  have exclusively very blue HB types, in this  $[\text{Fe}/\text{H}]$  range  $M_V(RR)$  is significantly brighter than the HB type = 0 case (the dashed line). As Figure 5 illustrates, this produces a very steep ( $\mu = 0.36$ )  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  relationship.

There is not a tight relationship between  $[\text{Fe}/\text{H}]$  and HB morphology among the globular clusters in the outer halo because of the second parameter effect. To obtain a sufficiently large sample of clusters to illustrate this, it is necessary to consider a wide range in  $R_{gc}$  because the number density of clusters falls off steeply with increasing  $R_{gc}$ . The clusters having  $6 < R_{gc} \leq 20 \text{ kpc}$  are plotted in the lower diagram of Fig. 4, where the diversity in HB types among the metal-poor clusters is obviously much larger than in the inner halo. Following the same procedures as before, we have estimated the mean HB types of the clusters at different values of  $[\text{Fe}/\text{H}]$  and have estimated values of  $M_V(RR)$  from our synthetic HB calculations. The resulting  $M_V(RR) - [\text{Fe}/\text{H}]$  relationship is shown in the lower diagram of Fig. 5. In contrast with the relationship for the inner halo, this one ( $\mu = 0.22$ ,  $\gamma = 0.90$ ) deviates only slightly from the case where HB type = 0 for all  $[\text{Fe}/\text{H}]$  (see Fig.3).

On the basis of Figs. 4 and 5, we conclude that a universal  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  relation does not exist because  $\langle M_V(RR) \rangle$  depends on HB morphology as well as  $[\text{Fe}/\text{H}]$  and because the relationship between HB morphology and  $[\text{Fe}/\text{H}]$  varies with the stellar population being considered. While this last point is best illustrated by the globular clusters in the inner and outer halo (Fig. 4), the recent work on the color-magnitude diagrams of the globular clusters in M31

(Fusi Pecci et al. 1996), M33 (Sarajedini et al. 1998), the LMC (Olsen et al. 1998), and the Fornax dwarf spheroidal galaxy (Buonanno et al. 1998) also show significant variations in the HB type -  $[\text{Fe}/\text{H}]$  relation from system to system. The data on these cluster systems, which for some of them is far from complete, suggest that the inner halo galactic may be the most extreme example where over a wide range of  $[\text{Fe}/\text{H}]$  only very blue HB types are found. Therefore, stellar populations in which  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  relation is as steep as the one for the inner halo clusters may be rare. Nonetheless, one should be cautious when adopting a  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  relation without information on the variation of HB type with  $[\text{Fe}/\text{H}]$  in the stellar population.

It is important to consider if the debate over the slope and zero-point of the  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  relation is partially due to the non-universality of this relation. The relation found here for the inner halo clusters resembles in slope the steep relationships that Sandage (1993 and references therein) obtained during his more than a decade long analyses of the Oosterhoff effect among the galactic globular clusters. We doubt, however, that they are related. Most of the clusters Sandage used in his analysis are rich in RR Lyrae variables and do not have extremely blue HBs. Our models predict that the  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  relation for these clusters is not steeply sloped. The  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  relation that Fusi Pecci et al. (1996) obtained from the color-magnitude diagrams of 8 globular clusters in M31 represents the other extreme, for they obtained the very shallow slope of  $\mu = 0.13 \pm 0.07$ . In only the 3 most metal poor clusters of this sample is the HB morphology sufficiently blue to populate the instability strip with RR Lyrae variables, and these clusters do not have very blue HB types in spite of  $[\text{Fe}/\text{H}] \sim -1.8$  to  $\sim -1.5$  (compare with the top diagram of Fig. 4). The other 5 clusters in this M31 sample have HB morphologies too red for RR Lyrae variables, and for them Fusi Pecci et al. had to resort to estimating the HB level at the instability strip from the observed red HB. Fusi Pecci et al. point out that their conclusion that  $\mu$  is small does not depend critically on this uncertain procedure. For this sample of M31 clusters, our calculations predict a  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  relation similar to the HB type = 0 case; hence  $\mu \sim 0.21$ . While this value is larger than the value obtained by Fusi Pecci et al. (1996), it is barely within one standard deviation of it. As Fusi Pecci et al. suggest, a larger sample of M31 clusters must be observed before one can be confident that the slope of the M31 relation is indeed incompatible with theoretical calculations such as ours. The zero-point  $\gamma$  of the M31 relation is fixed entirely by the distance adopted for M31, and the value obtained by Fusi Pecci et al. (1996) ( $0.95 \pm 0.09$ ) is consistent with our calculations.

Of course, the  $\langle M_V(RR) \rangle - [\text{Fe}/\text{H}]$  relation has also been investigated using samples of field RR Lyrae variables lying within a few kiloparsecs of the Sun. Some of these variables are members of the galactic halo, while others, preferentially the more metal-rich ones, are members of the thick disk population. Because it is difficult to recognize red HB stars in the field, the HB type -  $[\text{Fe}/\text{H}]$  relations of these populations are poorly known. The work by Preston, Sackett and Beers (1991) on the field HB stars and by Caputo (1993) on the RR Lyrae variables suggests that the HB morphology of the field may vary with  $R_{gc}$  in much the same way as the HB morphology of the globular clusters. Since a few metal-poor clusters with HB type  $< 0.9$  have values of  $R_{gc}$

that are not much different from the Sun's, we suspect the HB type -  $[\text{Fe}/\text{H}]$  relation of the field population near the Sun may resemble more the lower diagram of Fig. 4 than the upper. If this is correct, our models suggest that  $\mu \sim 0.22$ . This is close to the results obtained from applying the Baade-Wesselink method to samples of field RR Lyrae variables (see above). The values of  $\gamma$  from the Baade-Wesselink analyses, which are considered more susceptible to systematic error than  $\mu$ , are about 0.1 mag. fainter than the value given by our calculations (see Fernley et al. 1998).

In principle, the application of the method of statistical parallax to samples of field RR Lyrae variables should also yield the  $\langle M_V(RR) \rangle$ - $[\text{Fe}/\text{H}]$  relation. To date the samples of stars have proved inadequate for determining both  $\mu$  and  $\gamma$ , but precise values of  $\langle M_V(RR) \rangle$  have been obtained at the mean  $[\text{Fe}/\text{H}]$  of the sample of halo RR Lyrae variables. Recent results by Layden et al. (1996) and by Gould & Popowski (1998) have yielded  $M_V(RR)$  of  $+0.71 \pm 0.12$  and  $+0.77 \pm 0.13$  at  $[\text{Fe}/\text{H}] \sim -1.6$ , respectively. Our calculations give brighter values with the exact value depending on which HB type -  $[\text{Fe}/\text{H}]$  relation is adopted. For the outer halo one, which may be appropriate for the variables near the Sun,  $M_V(RR) = +0.55$  at  $[\text{Fe}/\text{H}] = -1.6$ , which slightly more than one sigma brighter than the results from statistical parallax. We have no explanation for this difference, which if confirmed, may mean that some revision of the models is required. While the dependence of  $M_V(RR)$  on HB morphology may not have a large effect on the either the Baade-Wesselink or the statistical parallax analyses, it is probably a contributing factor to the scatter in  $M_V(RR)$  among the metal-poor variables (see Jones et al. 1992; Fernley et al. 1998).

## 5. Discussion

The dependence of  $\langle M_V(RR) \rangle$  on HB morphology may have a significant effect upon astrophysical problems involving the distance scale for old stellar populations. We concentrate here on the question of the ages of globular clusters, which is important for both galactic evolution and for setting the minimum age of the universe. Our results suggest that assuming the same  $\langle M_V(RR) \rangle$ - $[\text{Fe}/\text{H}]$  relation for clusters of all HB types will significantly underestimate the distances to the metal-poor clusters having very blue HB morphologies. The luminosities of their main-sequence turnoffs will be underestimated and their ages will be overestimated. This effect is greatest for the clusters having the very bluest HB types ( $\sim 1.0$ ) (see also Storm et al. 1994; Clement et al. 1999).

To illustrate this, let us consider the globular cluster M92 (NGC 6341), which is one of the most metal poor globular clusters ( $[\text{Fe}/\text{H}] = -2.24$ , Zinn & West 1984) and perhaps one of the oldest. While M92 has a blue HB, it is not extremely blue (HB type = 0.88, Lee et al. 1994), and one might think that it is immune to this systematic error. According to our models, for this HB type  $\langle M_V(RR) \rangle$  is approximately 0.07 mag. brighter than the HB type = 0 case. Therefore, if the distance modulus of M92 is set using a  $\langle M_V(RR) \rangle$ - $[\text{Fe}/\text{H}]$  relation that is appropriate for HB types in the range -0.5 to +0.5, then the luminosity of its main-sequence turnoff will be underestimated by about 0.07 mag. This will cause its age to be overestimated by about 1 Gyr.



This  $\leq 10\%$  error in cluster age may appear small, it is nonetheless significant for either setting the lower limit on the age of the universe or for ascertaining the dispersion in age among the globular clusters and thereby distinguishing between different scenarios for the formation of the galactic halo.

The theoretical prediction (see also Lee et al. 1990, Caputo 1997, and refs. therein) that the RR Lyrae variables in M92 are highly evolved HB stars is consistent with the observational results of Storm et al. (1994) who measured the absolute magnitudes of two variables in M92 using the Baade-Wesselink technique. The mean value of  $\langle M_V(RR) \rangle$  that they obtained for these two stars is brighter than the  $\langle M_V(RR) \rangle$ -[Fe/H] relation they derived for field RR Lyrae variables by the same technique by  $0.21 \pm 0.15$  mag. While we predict a smaller offset, our result is within the errors of the value obtained from this very small sample.

Often observers will not use the few RR Lyrae variables in a blue HB cluster to set the apparent magnitude of the HB, but will use instead the reddest non-variable stars on the blue HB. SHB models (see fig. 1, 2, & 3 in Lee et al. 1990) indicate that these stars have also evolved far from the ZAHB, but somewhat less so than the RR Lyrae variables. At best, this practice will remove only partially the need to take into account HB morphology when estimating the distance modulus of a cluster. Care must also be exercised when turning this problem around and using a globular cluster of known distance to set the luminosity of RR Lyrae variables. For example, the globular cluster NGC 6752 has an extremely blue HB and lacks RR Lyrae variables. Its distance modulus has been measured from white dwarf fitting (Renzini et al. 1996). The absolute magnitude of the blue HB stars in NGC 6752 should not be assigned without correction to RR Lyrae variables of its metallicity.

Finally, we note that any detailed comparison between theory and observation is still hampered by empirical uncertainties on cluster distances as well as on individual cluster metallicities.

Partial support for this work was provided by NASA grant NAG5-8406 (P.D.), NSF grant AST-9319229 (R.Z.), and the Creative Research Initiative program of the Korean Ministry of Science & Technology (Y.-W.L. and S.Y.).

Table 1: Parameters used for the HB track construction.

$Z$	$Y$	Core Mass( $M_{\odot}$ )
0.0001	0.2356	0.5013
0.0004	0.2370	0.4981
0.0007	0.2390	0.4981
0.0010	0.2409	0.4901
0.0040	0.2416	0.4877

Table 2: The slope in the  $\langle M_V(RR) \rangle$ -[Fe/H] relation as a function of metallicity.

$Z$	$\mu$
0.0001 – 0.0004	0.154
0.0004 – 0.001	0.231
0.001 – 0.004	0.234

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# FIGURE CAPTIONS

Fig. 1.— Theoretical calibration, based on the SHB models, of  $M_V(RR)$  as a function of HB Type, for each metallicity.

Fig. 2.— Same as Figure 1, but showing the sensitivity of the models to the choice of  $\sigma$ .

Fig. 3.— For the case where HB type = 0, the mean absolute visual magnitudes  $\langle M_V(RR) \rangle$  of the RR Lyrae variables is plotted against  $[\text{Fe}/\text{H}]$ . The method of least squares was used to fit the straight line to the points. Its equation is:  $\langle M_V(RR) \rangle = 0.21[\text{Fe}/\text{H}] + 0.89$ .

Fig. 4.— For the globular clusters (open circles) in two ranges of galactocentric distance ( $R_{gc}$ ),  $[\text{Fe}/\text{H}]$  is plotted against HB type. The dashed line, which is a fit to the data in the upper diagram, is reproduced in the lower so that the differences between the two groups of clusters are more apparent. The x's mark the mean HB types used in the calculation of the values of  $\langle M_V(RR) \rangle$  (see text and Fig. 5).

Fig. 5.— These diagrams illustrate the differences between the  $\langle M_V(RR) \rangle$ - $[\text{Fe}/\text{H}]$  relations for the globular clusters in the inner and outer halos. The points represent our estimates of  $\langle M_V(RR) \rangle$  at the values of  $[\text{Fe}/\text{H}]$  and mean HB type (the x's in Fig. 4). The solid lines are fits by the method of least squares to the data points ( $\mu = 0.36$  and  $0.22$  and  $\gamma = 1.04$  and  $0.90$ , for the inner and outer halo relations, respectively). The dashed line in each diagram represents the case where HB type = 0.











